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WADD TECHNICAL NOTE 61-42

EFFECTS OF XLR-99 ENGINE NOZZLE OPTIMIZATION ON MAXIMUM ENGINE PERFORMANCE

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DIRECTORATE OF SYSTEMS ENGINEERING

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WRIGHT AIR DEVELOPMENT DIVISION

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Directorate of Systems Engineering

March 1961

Weapon System 653A

Wright Air Development Division
Air Research and Development Command
United States Air Force
Wright-Patterson Air Force Base, Ohio

FOREWORD

This report was prepared by Propulsion and Flight Vehicle Power Branch, Defensive and Experimental Systems Engineering Division, Directorate of Systems Engineering, Wright Air Development Division. The study was conducted under Weapon System 653A.

The studies presented began on 30 January 1961 and were concluded 20 February 1961.

ABSTRACT

The mission of the X-15 aircraft engine requires that the engine operate at 40,000 to 250,000 feet altitude. Since nozzle optimization is presently at 17,000 feet, methods of increasing engine performance by optimization of the nozzle to higher altitudes were studied. Theoretical results indicate that optimization can be achieved at 40,000 feet by extending the present nozzle 18 inches. The exit area of this nozzle would still be within the limits imposed by the contour of the X-15 tail section. Optimization over 40,000 feet would probably require major design changes.

PUBLICATION REVIEW

This report has been reviewed and is approved.

FOR THE COMMANDER:

James G. Barrett

JAMES G. BARRETT
Chief, Propulsion and Flight Vehicle Power Branch
Defensive and Experimental Systems Engineering Division
Directorate of Systems Engineering

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LIST OF SYMBOLS

A_t	=	nozzle throat area (in^2)
A_e	=	nozzle exit area (in^2)
$\frac{A_e}{A_t}$	=	nozzle area ratio
P_e	=	absolute pressure at nozzle exit (lbs per ft^2)
P_c	=	absolute pressure in thrust chamber (lbs per ft^2)
P_a	=	ambient pressure (lbs per ft^2)
C_{fvac}	=	theoretical vacuum thrust coefficient
C_{fth}	=	C_{fvac} corrected for altitude
C_{fact}	=	C_{fth} corrected for the nozzle discharge coefficient and the nozzle efficiency factor. (For the XLR-99 nozzle C_{fact} = 0.934 C_{fth}) (approx.)
$\dot{W}$	=	weight flow rate of propellant (lbs per sec)
L	=	length of nozzle exit cone (in.)
F_{th}	=	theoretical thrust (C_{fth} P_c A_t)
F_{ac}	=	delivered thrust (C_{fact} P_c A_t)
I_{spth}	=	theoretical specific impulse $\frac{(F_{th})}{\dot{W}}$
I_{spact}	=	delivered specific Impulse $\frac{(F_{act})}{\dot{W}}$
k	=	ratio of specific heats

INTRODUCTION

The mission of the XLR-99 rocket engine in the X-15 aircraft requires that the engine operate at altitudes ranging from 40,000 to 250,000 feet. At present the optimum performance of the XLR-99 engine nozzle is at 17,000 feet altitude, which is considerably under the minimum operating altitude of the engine. The objective of our efforts, therefore, was to determine how nozzle optimization could be achieved with only minor design changes.

Our approach was to determine if the nozzle could be extended and still maintain its exit area within the limits imposed by the present contour of the X-15 tail section. Structural design, added inert weight, center-of-gravity shift, and increased drag associated with optimization of the XLR-99 engine nozzle to a typical mission altitude were assumed to be more important criteria than the increased performance obtained from the engine. Our study was based on theory, and calculations were obtained from the theory applied. The effects of optimization of the nozzle on the maximum thrust and specific impulse of the XLR-99 rocket engine were determined. The effects of increasing drag and added inert weight with nozzle extensions to the engine were not calculated.

Manuscript released by author on 28 February 1961 for publication as a WADD Technical Note.

THEORETICAL APPROACH

The following two methods were used to calculate nozzle optimization.

Method 1

a. Choose a given altitude for nozzle optimization.

b. Determine P_a at this altitude from I.C.A.O. standard atmosphere chart (Reference 1).

c. For optimum expansion, set $P_a = P_e$.

d. Determine the ratio of $\frac{P_e}{P_c}$ for a given P_c.

e. Calculate the optimum expansion ratio by the following method (Reference 2):

$$\frac{A_t}{A_e} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_e}{P_c}\right)^{\frac{1}{k}} \sqrt{\left(\frac{k+1}{k-1}\right)\left[1-\left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}}\right]} \cdot$$

f. Calculate C_{fvac} as follows (Reference 2):

$$C_{fvac} = \sqrt{\frac{2k^2}{k-1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \left[1-\left(\frac{P_e}{P_c}\right)^{\frac{k-1}{k}}\right]} + \left(\frac{P_e}{P_c}\right)\left(\frac{A_e}{A_t}\right) \cdot$$

g. Calculate C_{fth} as follows:

$$C_{fth} = C_{fvac} - \left(\frac{A_e}{A_t}\right)\left(\frac{P_a}{P_c}\right) \cdot$$

h. Calculate C_{fact} as follows:

Overall nozzle efficiency as determined experimentally for the XLR-99 engine is equal to 0.934; therefore, $C_{fact} = 0.934\ C_{fth}$.

i. Calculate other performance parameters as follows:

$$F_{th} = C_{fth}\ P_c\ A_t$$

$$F_{act} = C_{fact}\ P_c\ A_t$$

$$I_{spth} = \frac{F_{th}}{\dot{W}}$$

$$I_{spact} = \frac{F_{act}}{\dot{W}} .$$

Method 2

a. Choose a given area ratio.

b. Using the given area ratio, solve equation given as part 1 of Method 1 for $\frac{P_e}{P_c}$.

c. From $\frac{P_e}{P_c}$, solve for P_e using a given P_c.

d. Set $P_e = P_a$ for optimum expansion.

e. From I.C.A.O. standard atmosphere chart (Reference 1), determine the altitude of nozzle optimization.

f. Same as part f of method 1.

g. Same as part g of method 1.

h. Same as part h of method 1.

i. Same as part i of method 1.

RESULTS

The detailed results of optimization of the conical nozzle are given in the appendix. From these data, Figures 1 through 4 were plotted. These figures show that the XLR-99 rocket engine operates more efficiently when nozzle optimization is at the altitude at which the engine is to operate. By extending the present nozzle 18 inches, we can obtain nozzle optimization at 40,000 feet altitude. The exit area of this extended nozzle would still be within the limits imposed by the present contour of the X-15 tail section. This degree of extension is about the limit obtainable with only minor design changes. As shown in the appendix, optimization of this nozzle to greater altitudes requires nozzle extensions that are prohibitive in size.

The rate of increase in thrust and specific impulse decreases with optimization of the nozzle to increasing altitudes. At the same time, the rate of increase in size of the nozzle increases with nozzle optimization to increasing altitude. If increasing the performance of the engine about 4 to 5 percent is desired, nozzle optimization to 40,000 feet altitude would probably be the most feasible method. Optimization of this nozzle to altitudes greater than 40,000 feet would probably cause major design and stabilization problems.

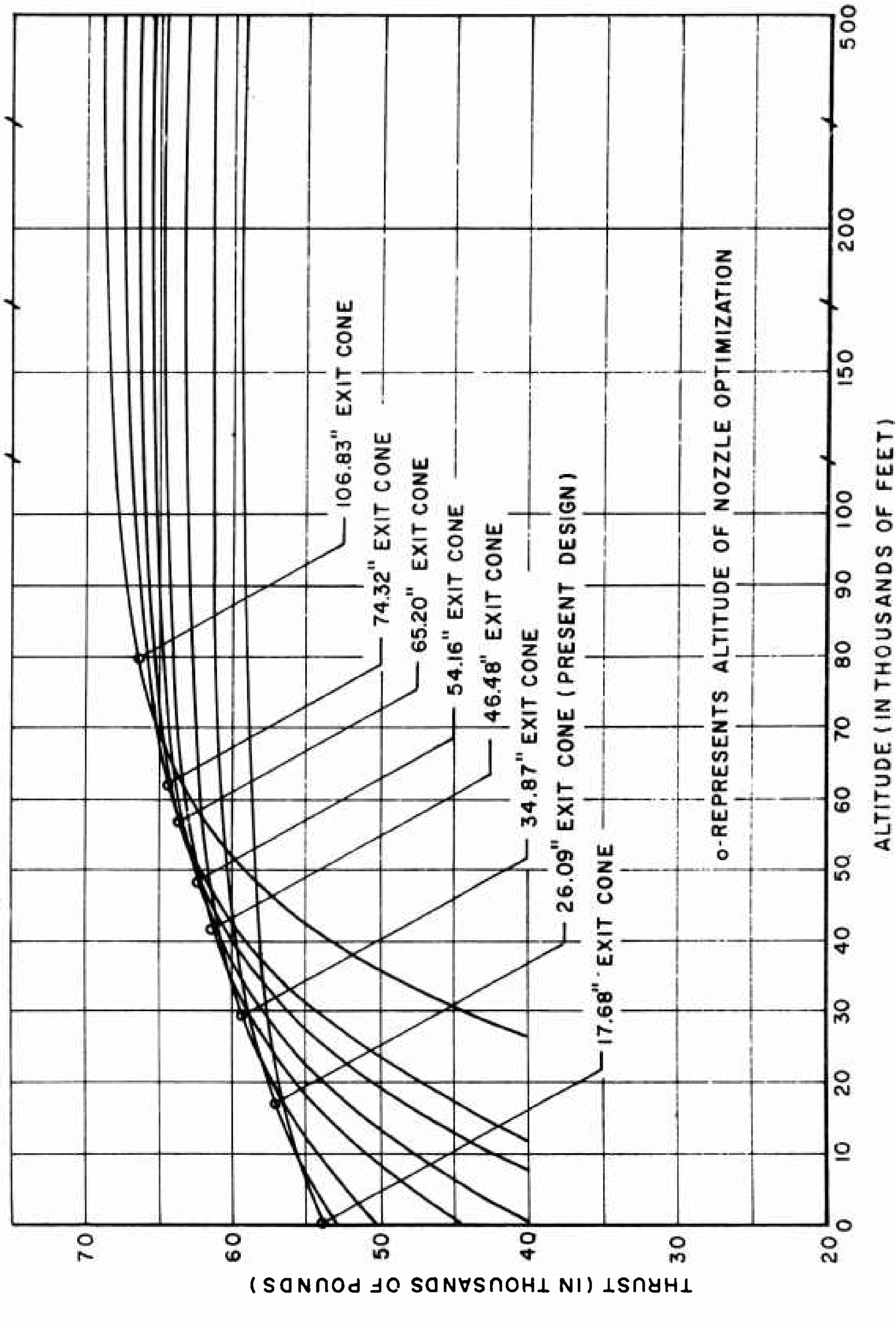

Figure 1. Altitude Vs Theoretical Thrust for Various Nozzle Extensions

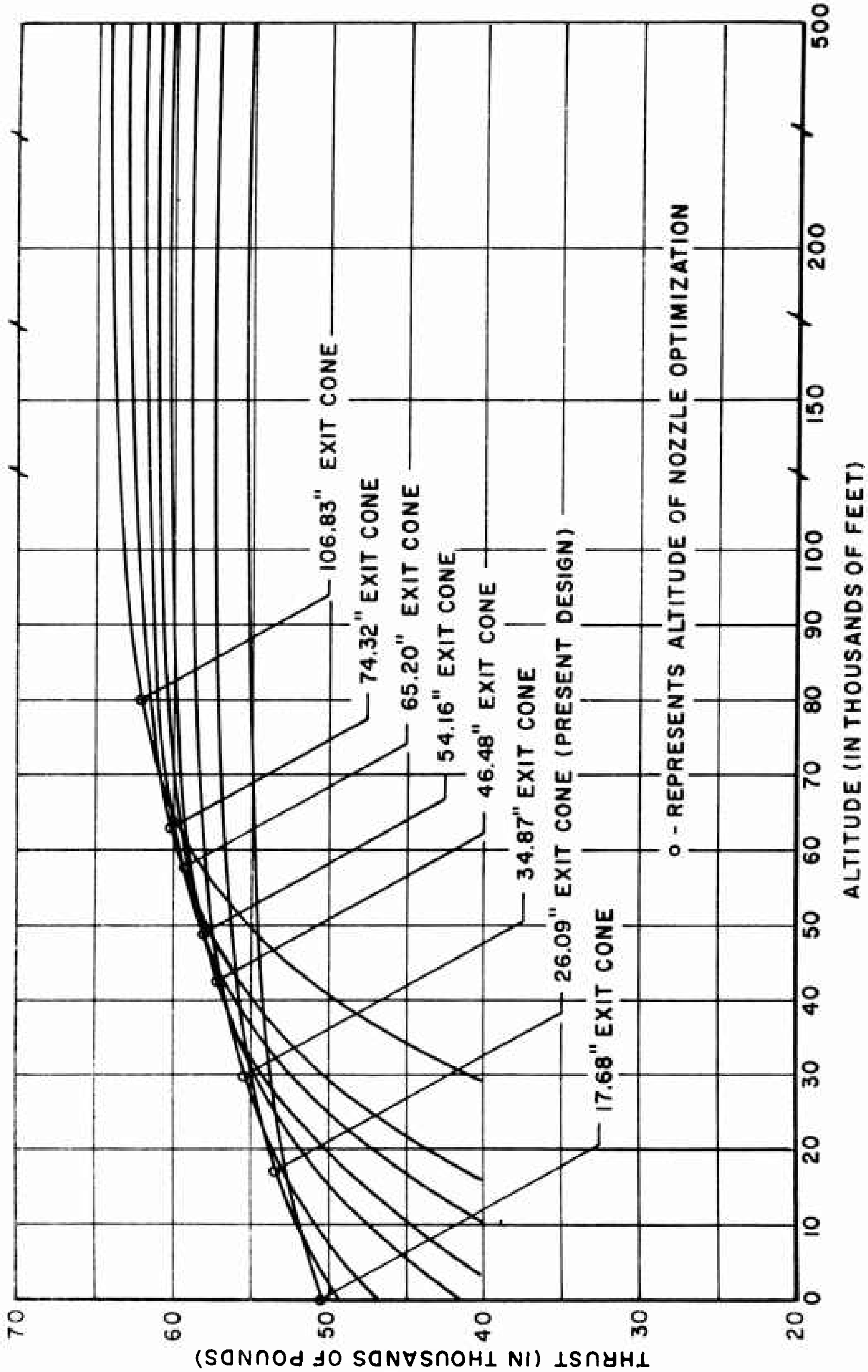

Figure 2. Altitude Vs Actual Thrust for Various Nozzle Extensions

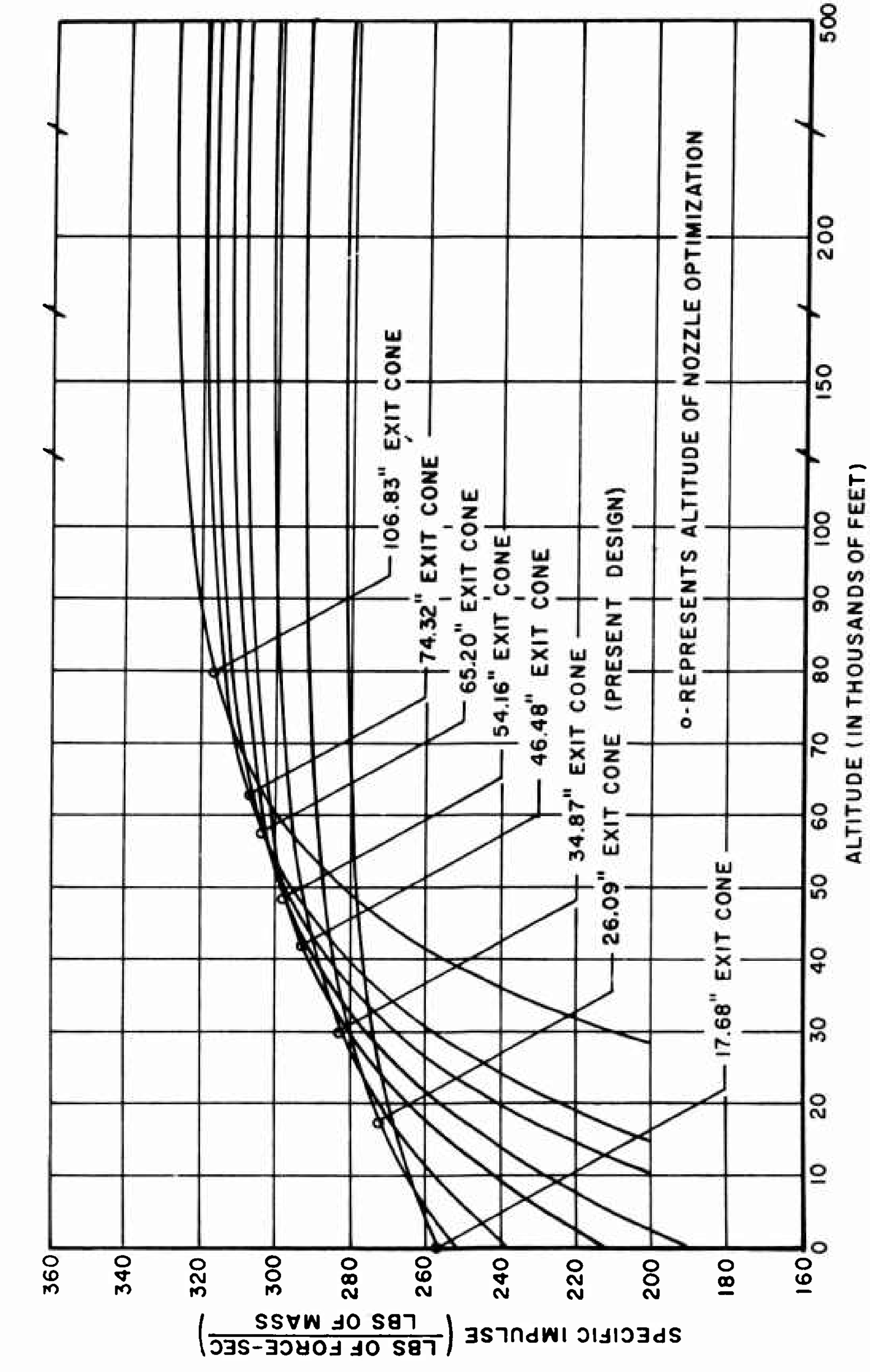

Figure 3. Altitude Vs Theoretical Specific Impulse for Various Nozzle Extensions

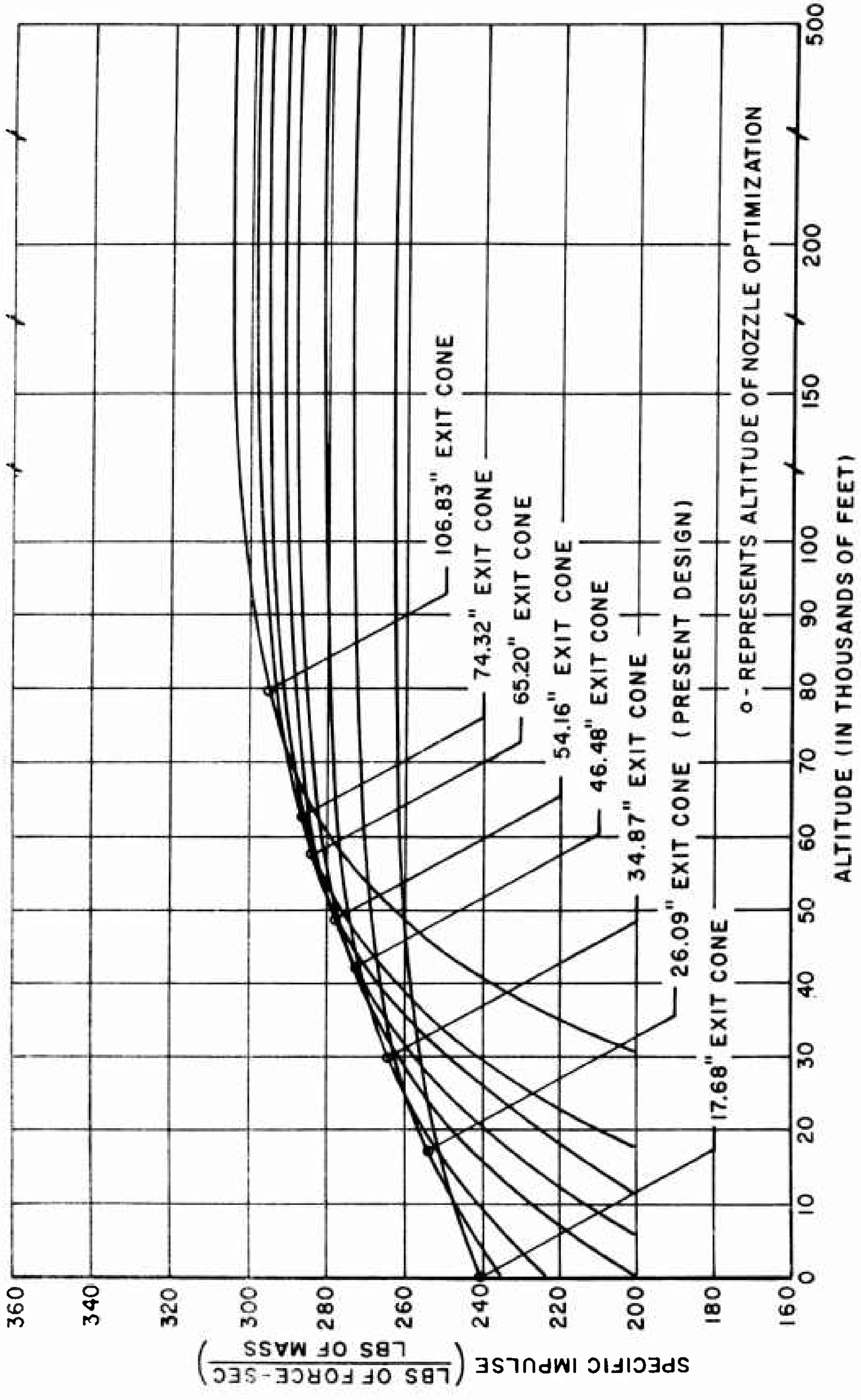

Figure 4. Altitude Vs Actual Specific Impulse for Various Nozzle Extensions

APPENDIX

DATA ON NOZZLE OPTIMIZATION FROM SEA LEVEL TO 80,000 FEET ALTITUDE

0 Feet Altitude

A_t 59.038 in^2

A_e 352.61 in^2

$\frac{A_e}{A_t}$ 5.97258

$\frac{P_e}{P_c}$ 0.02449

C_{fvac} 1.66804

P_c = 86,400 lbs per ft^2

P_e = 2116.22 lbs per ft^2

k = 1.21

$\dot{W}_{max}$ = 209.7 lbs per sec

L = 17.68 in.

Altitude	C_{fth}	F_{th}	F_{act}	I_{spth}	I_{spact}	P_a
0	1.52176	53,905.0	50,347.3	257.13	240.16	2116.22
10,000	1.56744	55,523.1	51,858.6	264.84	247.36	1455.33
20,000	1.60082	56,705.5	52,962.9	270.48	252.63	972.49
30,000	1.62460	57,547.9	53,749.7	274.50	256.38	628.43
40,000	1.64097	58,127.8	54,291.4	277.27	258.97	391.68
50,000	1.65130	58,493.7	54,633.1	279.01	260.59	242.21
60,000	1.65769	58,720.0	54,844.5	280.09	261.60	149.78
70,000	1.66158	58,857.8	54,973.2	280.75	262.22	93.52
80,000	1.66403	58,944.6	55,054.2	281.16	262.60	58.01
90,000	1.66556	58,998.8	55,104.9	281.42	262.85	35.95
100,000	1.66650	59,032.1	55,136.0	281.58	262.99	22.31
150,000	1.66784	59,079.6	55,180.3	281.81	263.21	3.00
200,000	1.66800	59,085.2	55,185.6	281.84	263.24	0.66
500,000	1.66804	59,086.6	55,186.9	281.84	263.24	0.00

17,345 Feet Altitude

A_t 59.038 in^2 | P_c = 86,400 lbs per ft^2

A_e 578.572 in^2 | P_e = 1085.84 lbs per ft^2

$\frac{A_e}{A_t}$ 9.800 | k = 1.21

$\frac{P_e}{P_c}$ 0.0125676 | $\dot{W}_{max}$ = 209.7 lbs per sec

C_{fvac} 1.7337921 | L = 26.09 in.

Altitude	C_{fth}	F_{th}	F_{act}	I_{spth}	I_{spact}	P_a
0	1.49376	52,913.2	49,420.9	252.40	235.74	2116.22
10,000	1.56872	55,568.5	51,901.0	265.06	247.57	1455.33
20,000	1.62349	57,508.6	53,713.0	274.32	256.21	972.49
17,345	1.61063	57,053.0	53,287.5	272.14	254.18	1085.84
30,000	1.66251	58,890.8	55,004.0	280.91	262.37	628.43
40,000	1.68937	59,842.2	55,892.6	285.45	266.61	391.68
50,000	1.70632	60,442.6	56,453.4	288.31	269.28	242.21
60,000	1.71681	60,814.2	56,800.5	290.08	270.93	149.78
70,000	1.72319	61,040.2	57,011.5	291.16	271.94	93.52
80,000	1.72722	61,183.0	57,144.9	291.84	272.58	58.01
90,000	1.72972	61,271.5	57,227.6	292.26	272.97	35.95
100,000	1.73126	61,326.1	57,278.6	292.52	273.21	22.31
150,000	1.73345	61,403.6	57,351.0	292.89	273.56	3.00
200,000	1.73372	61,413.2	57,359.9	292.94	273.60	0.66
500,000	1.73379	61,415.7	57,362.3	292.95	273.60	0.00

30,000 Feet Altitude

A_t	59.038 in^2		P_c	= 86,400 lbs per ft^2
A_e	874.35 in^2		P_e	= 628.431 lbs per ft^2
$\frac{A_e}{A_t}$	14.809		k	= 1.21
$\frac{P_e}{P_c}$	0.00727		$\dot{W}_{max}$	= 209.7 lbs per sec
C_{fvac}	1.78132		L	= 34.87 in.

Altitude	C_{fth}	F_{th}	F_{act}	I_{spth}	I_{spact}	P_a
0	1.41860	50,250.8	46,934.2	239.70	223.88	2116.22
10,000	1.52188	54,263.5	50,682.1	258.84	241.76	1455.33
20,000	1.61464	57,195.1	53,420.2	272.82	254.81	972.49
30,000	1.67361	59,283.9	55,371.2	282.78	264.12	628.43
40,000	1.71419	60,721.4	56,713.8	289.64	270.52	391.68
50,000	1.73981	61,628.9	57,561.4	293.97	274.57	242.21
60,000	1.75565	62,190.0	58,085.5	296.65	277.07	149.78
70,000	1.76530	62,531.9	58,404.8	298.28	278.59	93.52
80,000	1.77138	62,747.2	58,605.9	299.30	279.55	58.01
90,000	1.77516	62,881.1	58,730.9	299.94	280.14	35.95
100,000	1.77750	62,964.0	58,808.4	300.34	280.52	22.31
150,000	1.78081	63,081.3	58,917.9	300.90	281.04	3.00
200,000	1.78121	63,095.4	58,931.1	300.96	281.10	0.66
500,000	1.78132	63,099.3	58,934.7	300.98	281.11	0.00

42,144 Feet Altitude

A_t	59.038 in^2	P_c	= 86,400 lbs per ft^2
A_e	1357.87 in^2	P_e	= 353.38 lbs per ft^2
$\frac{A_e}{A_t}$	23	k	= 1.21
$\frac{P_e}{P_c}$	0.0040907	$\dot{W}_{max}$	= 209.7 lbs per sec
C_{fvac}	1.825664	L	= 46.48 in.

Altitude	C_{fth}	F_{th}	F_{act}	I_{spth}	I_{spact}	P_a
0	1.26239	44,717.4	41,766.0	213.30	199.22	2116.22
10,000	1.43834	50,950.0	47,587.3	243.03	226.99	1455.33
20,000	1.56691	55,504.3	51,841.0	264.75	247.28	972.49
30,000	1.65845	58,746.9	54,869.6	280.22	261.72	628.43
40,000	1.72147	60,979.3	56,954.7	290.87	271.67	391.68
42,144	1.73159	61,337.8	57,289.5	292.58	273.27	353.38
50,000	1.76126	62,388.8	58,271.1	297.59	277.95	242.21
60,000	1.78587	63,260.5	59,085.3	301.75	281.83	149.78
70,000	1.80082	63,790.0	59,579.9	304.28	284.20	93.52
80,000	1.81025	64,124.1	59,891.9	305.87	285.68	58.01
90,000	1.81623	64,335.9	60,089.7	306.88	286.62	35.95
100,000	1.81991	64,466.3	60,211.5	307.50	287.20	22.31
150,000	1.82497	64,645.5	60,378.9	308.36	288.01	3.00
200,000	1.82566	64,670.0	60,401.8	308.48	288.12	0.66
500,000	1.82566	64,670.0	60,401.8	308.48	288.12	0.00

48,733 Feet Altitude

A_t	59.038 in^2	P_c	= 86,400 lbs per ft^2
A_e	1735.72 in^2	P_e	= 257.47 lbs per ft^2
$\frac{A_e}{A_t}$	29.4	k	= 1.21
$\frac{P_e}{P_c}$	0.0029803	$\dot{W}_{max}$	= 209.7 lbs per sec
C_{fvac}	1.84794	L	= 54.16 in.

Altitude	C_{fth}	F_{th}	F_{act}	I_{spth}	I_{spact}	P_a
0	1.12793	39,954.4	37,317.4	190.58	178.00	2116.22
10,000	1.35284	47,921.4	44,758.6	228.58	213.49	1455.33
20,000	1.51719	53,743.1	50,196.0	256.35	239.43	972.49
30,000	1.63420	57,887.9	54,067.3	276.12	257.90	628.43
40,000	1.71476	60,741.6	56,732.6	289.74	270.62	391.68
48,733	1.76062	62,366.1	58,250.0	297.49	277.85	257.47
50,000	1.76562	62,543.2	58,415.3	298.33	278.64	242.21
60,000	1.79708	63,657.6	59,456.2	303.65	283.61	149.78
70,000	1.81619	64,334.5	60,088.4	306.87	286.62	93.52
80,000	1.82824	64,761.4	60,487.1	308.91	288.52	58.01
90,000	1.83589	65,032.4	60,740.3	310.20	289.73	35.95
100,000	1.84059	65,198.8	60,895.7	311.00	290.47	22.31
150,000	1.84706	65,428.0	61,109.7	312.09	291.49	3.00
200,000	1.84794	65,459.2	61,138.9	312.24	291.63	0.66
500,000	1.84794	65,459.2	61,138.9	312.24	291.63	0.00

57,924 Feet Altitude

A_t	59.038 in^2	P_c	=	86,400 lbs per ft^2
A_e	2361.52 in^2	P_e	=	173.66 lbs per ft^2
$\frac{A_e}{A_t}$	40.0	k	=	1.21
$\frac{P_e}{P_c}$	0.0020064	$\dot{W}_{max}$	=	209.7 lbs per sec
C_{fvac}	1.8737727	L	=	65.20 in.

Altitude	C_{fth}	F_{th}	F_{act}	I_{spth}	I_{spact}	P_a
0	0.89404	———	———	———	———	2116.22
10,000	1.20001	42,507.7	39,702.2	203.05	189.65	1455.33
20,000	1.42355	50,426.1	47,098.0	240.53	224.65	972.49
30,000	1.58284	56,068.6	52,368.1	267.45	249.80	628.43
40,000	1.69244	59,951.0	55,994.2	285.97	267.09	391.68
50,000	1.76164	62,402.2	58,283.6	297.66	278.01	242.21
57,924	1.79338	63,526.5	59,333.7	303.02	283.02	173.66
60,000	1.80443	63,918.0	59,699.4	304.89	284.77	149.78
70,000	1.83048	64,840.7	60,561.2	309.29	288.88	93.52
80,000	1.84692	65,423.1	61,105.2	312.07	291.47	58.01
90,000	1.85713	65,784.7	61,442.9	313.79	293.08	35.95
100,000	1.86345	66,008.6	61,652.0	314.86	294.08	22.31
150,000	1.87239	66,325.3	61,947.8	316.37	295.49	3.00
200,000	1.87347	66,363.5	61,983.5	316.55	295.66	0.66
500,000	1.87377	66,374.2	61,993.5	316.60	295.70	0.00

62,876 Feet Altitude

A_t	59.038 in^2	P_c	= 86,400 lbs per ft^2
A_e	2951.9 in^2	P_e	= 130.46 lbs per ft^2
$\frac{A_e}{A_t}$	50	k	= 1.21
$\frac{P_e}{P_c}$	0.0015093	$\dot{W}_{max}$	= 209.7 lbs per sec
C_{fvac}	1.8911427	L	= 74.32 in.

Altitude	C_{fth}	F_{th}	F_{act}	I_{spth}	I_{spact}	P_a
0	0.66648	———	———	———	———	2116.22
10,000	1.04894	37,156.4	34,704.1	177.24	165.54	1455.33
20,000	1.32836	47,054.2	43,948.6	224.45	209.64	972.49
30,000	1.52747	54,107.3	50,536.2	258.09	241.06	628.43
40,000	1.66448	58,960.5	55,069.1	281.24	262.68	391.68
50,000	1.75098	62,024.6	57,931.0	295.86	276.33	242.21
60,000	1.80447	63,919.4	59,700.7	304.89	284.77	149.78
62,876	1.81565	64,315.4	60,070.6	306.78	286.53	130.46
70,000	1.83702	65,072.4	60,777.6	310.39	289.90	93.52
80,000	1.85757	65,800.3	61,457.5	313.87	293.15	58.01
90,000	1.87034	66,252.7	61,880.0	316.02	295.16	35.95
100,000	1.87823	66,532.2	62,141.1	317.36	296.41	22.31
150,000	1.88941	66,928.2	62,510.9	319.25	298.18	3.00
200,000	1.89076	66,976.0	62,555.6	319.47	298.38	0.66
500,000	1.89114	66,989.5	62,568.2	319.54	298.45	0.00

80,000 Feet Altitude

A_t	59.038 in^2	P_c	= 86,400 lbs per ft^2
A_e	5586.6 in^2	P_e	= 58.01 lbs per ft^2
$\frac{A_e}{A_t}$	94.627	k	= 1.21
$\frac{P_e}{P_c}$	0.00067	$\dot{W}_{max}$	= 209.7 lbs per sec
C_{fvac}	1.93536	L	= 106.83 in.

Altitude	C_{fth}	F_{th}	F_{act}	I_{spth}	I_{spact}	P_a
0	——	——	——	——	——	2116.22
10,000	0.34146	——	——	——	——	1455.33
20,000	0.87027	——	——	——	——	972.49
30,000	1.24710	44,175.8	41,260.2	210.72	196.81	628.43
40,000	1.50639	53,360.5	49,838.7	254.53	237.73	391.68
50,000	1.67010	59,159.6	55,255.1	282.19	263.56	242.21
60,000	1.77132	62,745.1	58,603.9	299.29	279.54	149.78
70,000	1.83294	64,927.9	60,642.6	309.71	289.27	93.52
80,000	1.87183	66,305.4	61,929.2	316.28	295.40	58.01
90,000	1.89599	67,161.3	62,728.6	320.36	299.22	35.95
100,000	1.91093	67,690.5	63,222.9	322.88	301.57	22.31
150,000	1.93208	68,439.7	63,922.7	326.46	304.91	3.00
200,000	1.93464	68,530.4	64,007.4	326.89	305.31	0.66
500,000	1.93536	68,555.9	64,031.2	327.01	305.43	0.00

LIST OF REFERENCES

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